James Binney (Oxford)

Building a non-axisymmetric Galaxy

Outline

- Equilibrium models
- Torus mapping
- Resonant trapping
- Non-axisymmetric tori
- Bar trapping and the UV plane

Equilibrium models vital

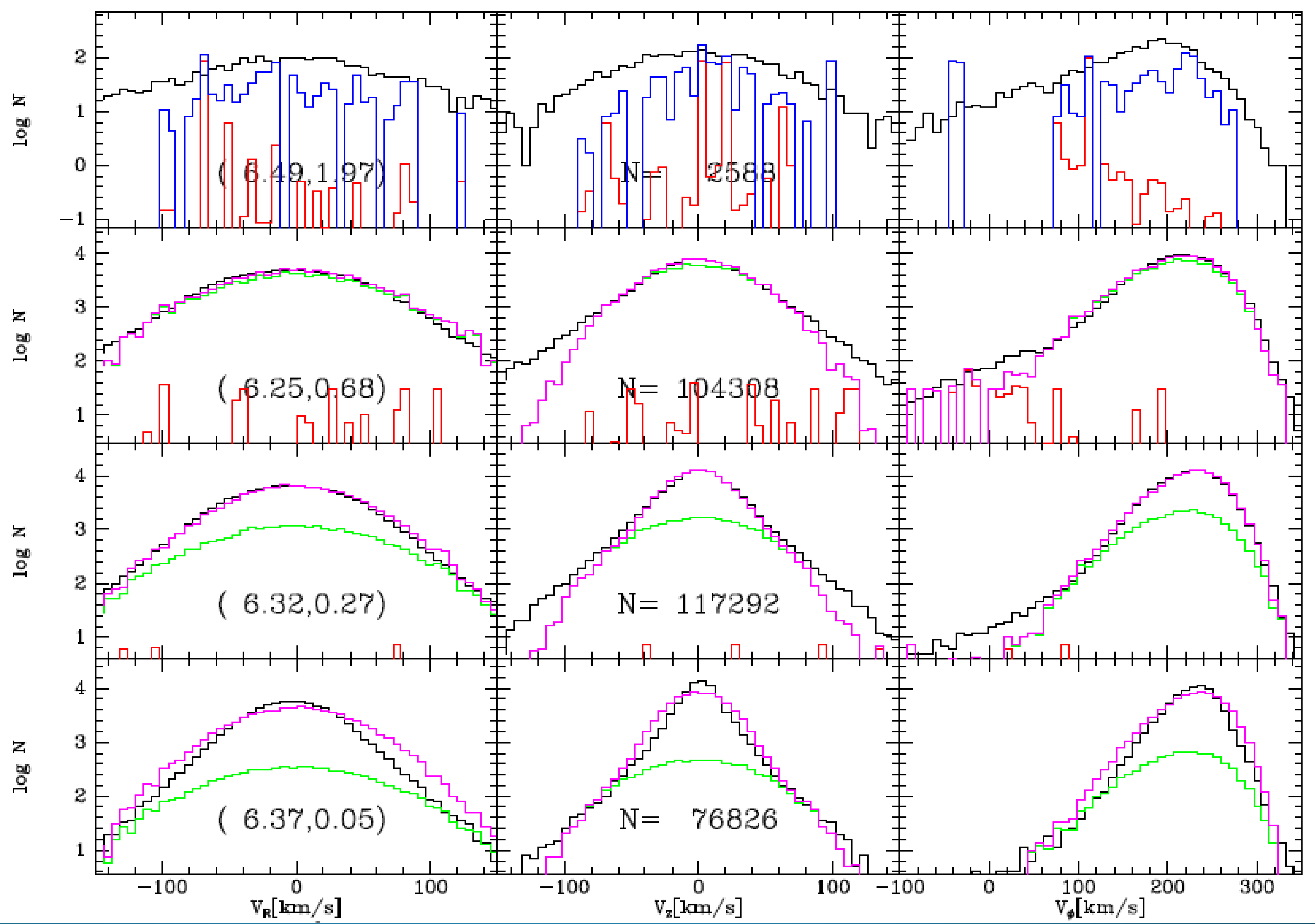
- Only tool to map DM
- Perfect way to start an N-body simulation
- Perfect tool to organise knowledge:
  - f(J,age,chemistry) + selection fn -> expectation of any survey query (Besancon model)

Modelling technologies

- M2M
  - Syer & Tremaine 1996 -> Morganti+2013, Portail+2015,..
- f(J)
  - B (2010), B (2012), B (2014), Piffl+ (2014,2015), Post+(2017), Pascale+(2019)
  - For each component (incl DM) choose analytic f(J,age,..)
  - Mass of each component determined up front
  - Solve for self-consistently generated $\Phi(x)$
  - Then any observable can be predicted
  - Fully exploit Jeans thm

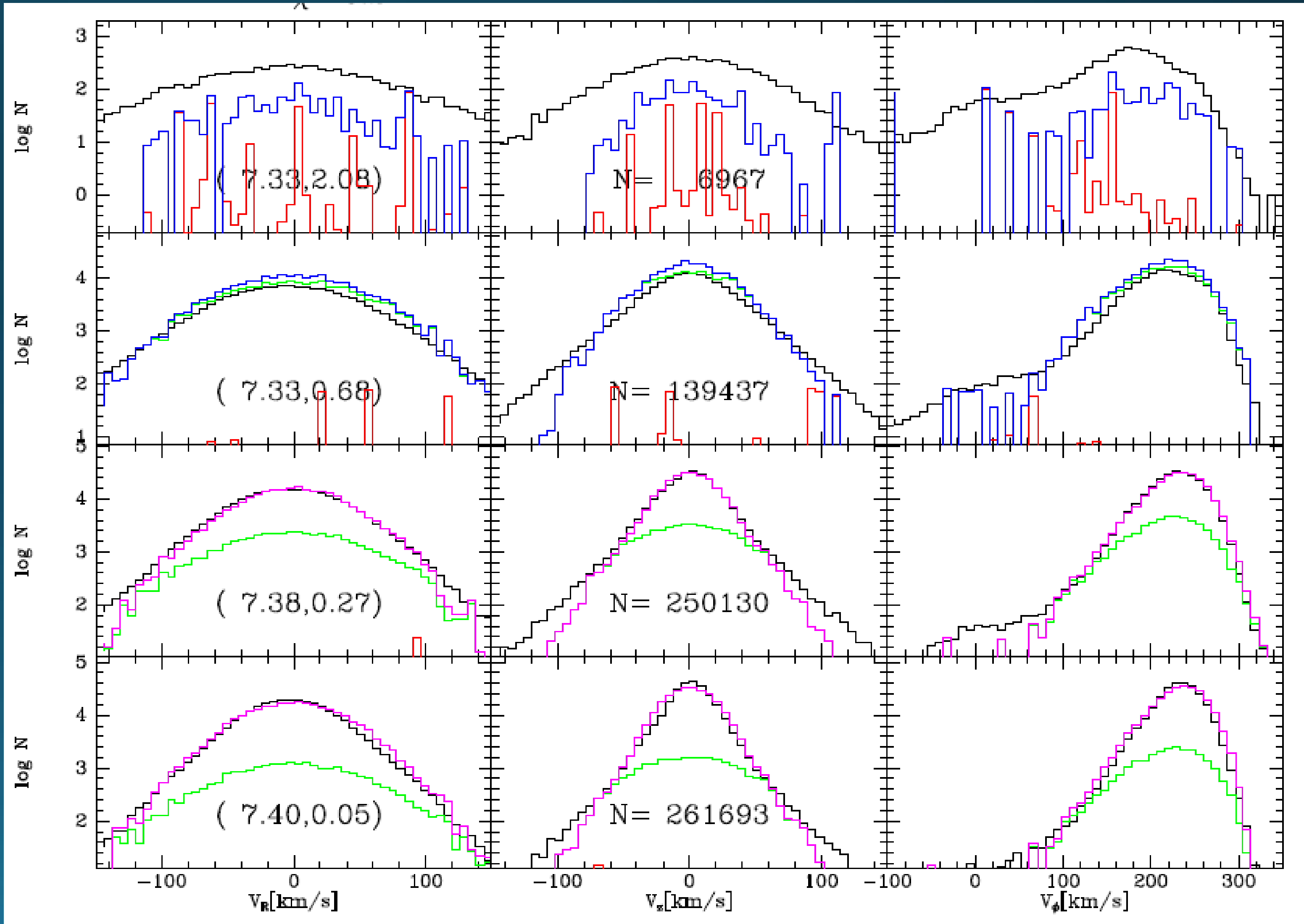
Axisymmetric model fitted to Gaia DR2

- Red: *halo
- Green: *halo+thk disc
- Blue: total
- Black: data
- R~6.3 kpc



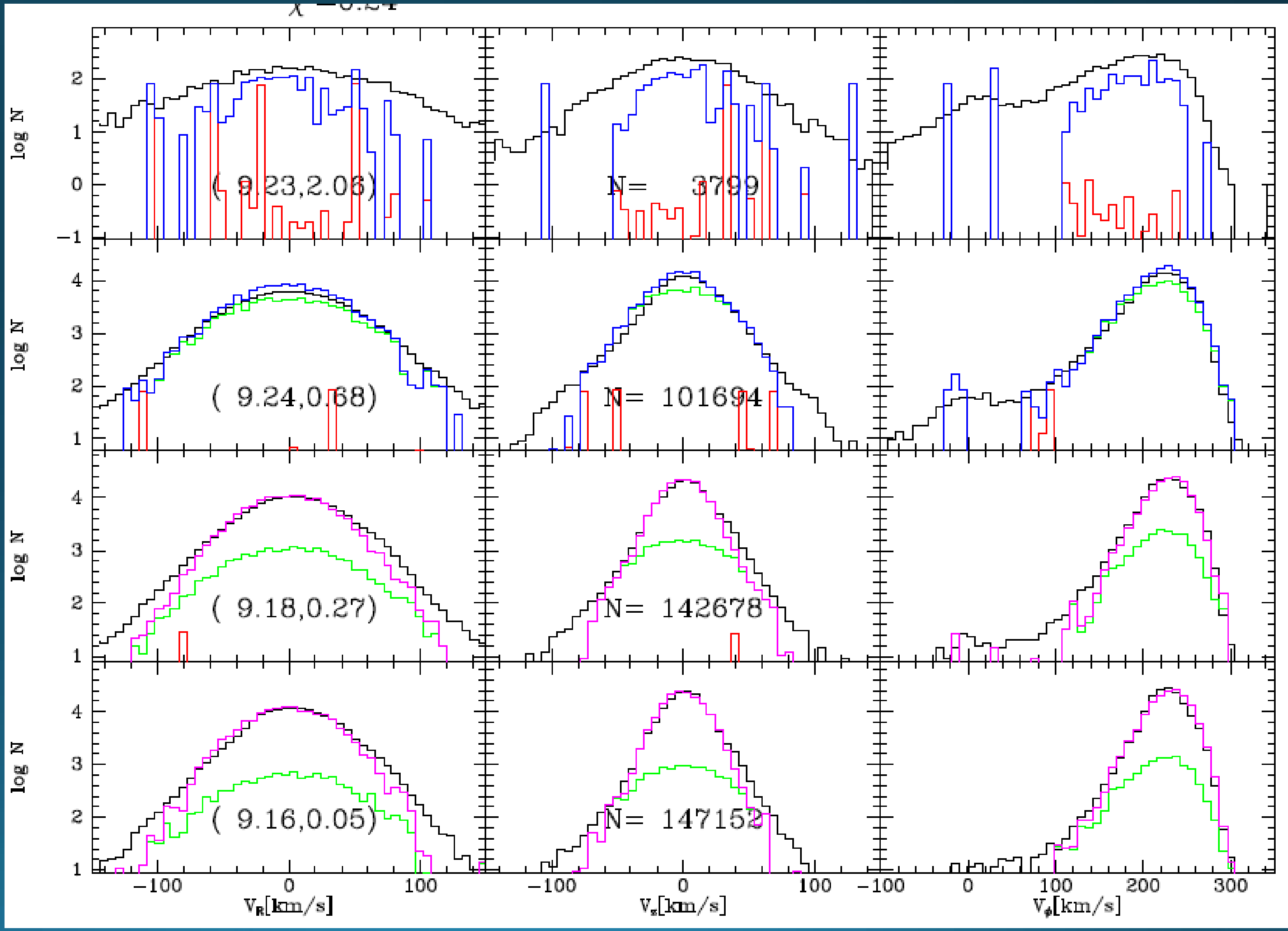
Model fitted to Gaia DR2 (R~7.4)

- Red: *halo
- Green: *halo+thk disc
- Blue: total
- Black: data



Model fitted to Gaia DR2 (R~9.2)

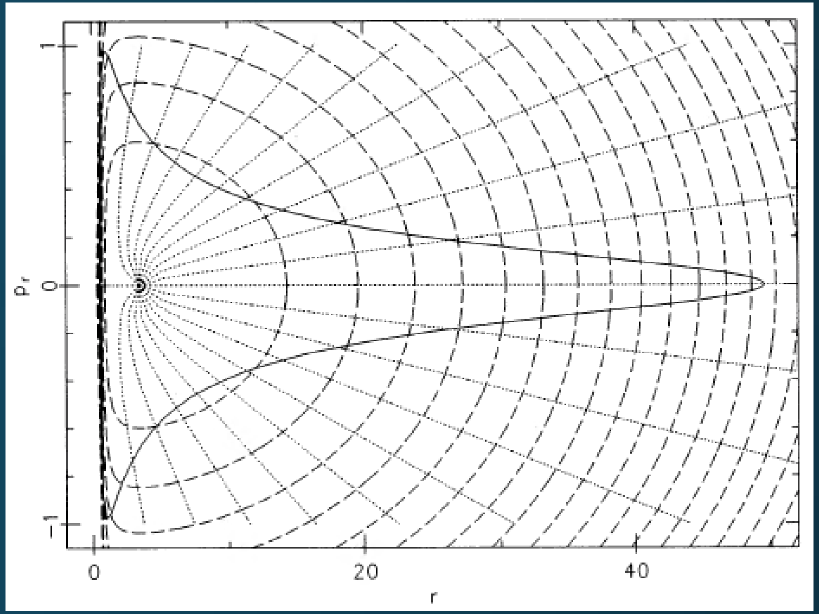
- Red: *halo
- Green: *halo+thk disc
- Blue: total
- Black: data



Getting & using AA-variables

- Model just shown uses the Staeckel Fudge (B 2012) to pass (x,v) -> (J,θ)
- Annie remarked that Besancon model fits Staeckel Φ(x) to real Φ(x)
- Staeckel Fudge applies to any reasonable axisymmetric Φ(x) and triaxial Φ(x) without figure rotation (Sanders & B 2014)
- But a rotating bar is out of scope

Torus mapping

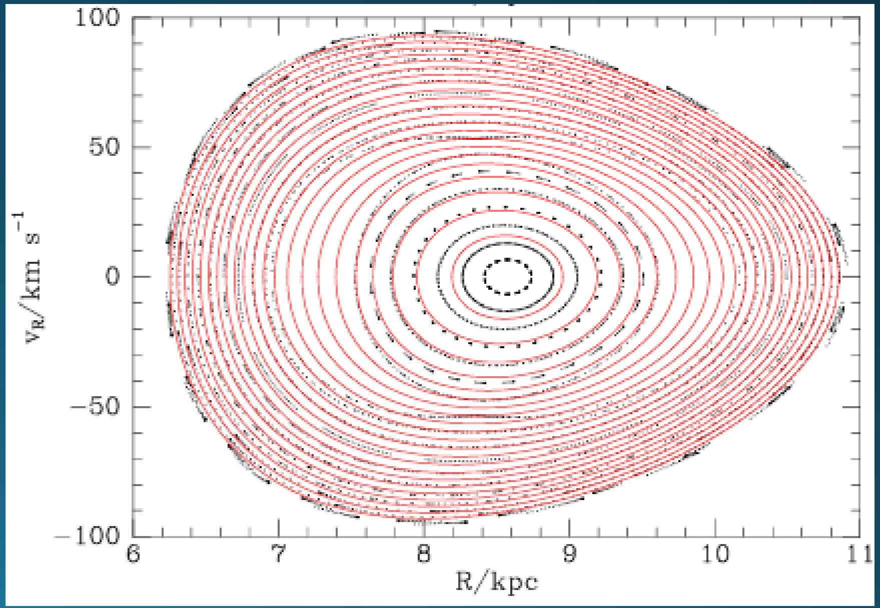


McGill & B 1990

- Regular orbits cover 3-tori in 6d phase space
- They lie within 5d hyperspheres H(x,v)=E
- The tori are null:
  - Poincare invariant of any 2d surface within them vanishes
  - Poincare invariant to Hamiltonian mech what $g_{\mu\nu}$ is to relativity
- If a null 3-torus lies within H=const, it's an orbital torus!
- Image of a null torus under canonical map is null
- So we find orbital tori by projecting toy torus into phase space & adjusting parameters of map until H~const on image (McGill & B 1990, Kaasalainen & B 1995)
- Think of this as upgrade to Runge-Kutta routine

Torus mapping

- Actions of torus inherited from toy torus
- Can construct conjugate angle variables for image
- After constructing grid of tori, fill gaps by interpolation
- Then have system of AA vars (θ,J)



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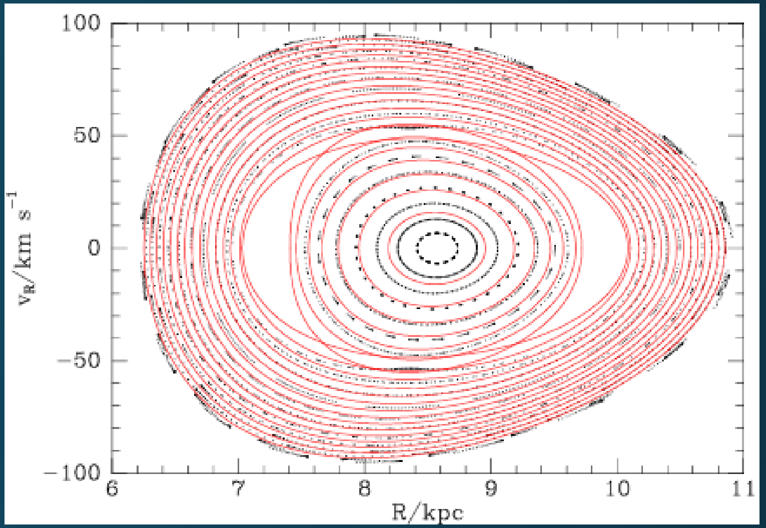
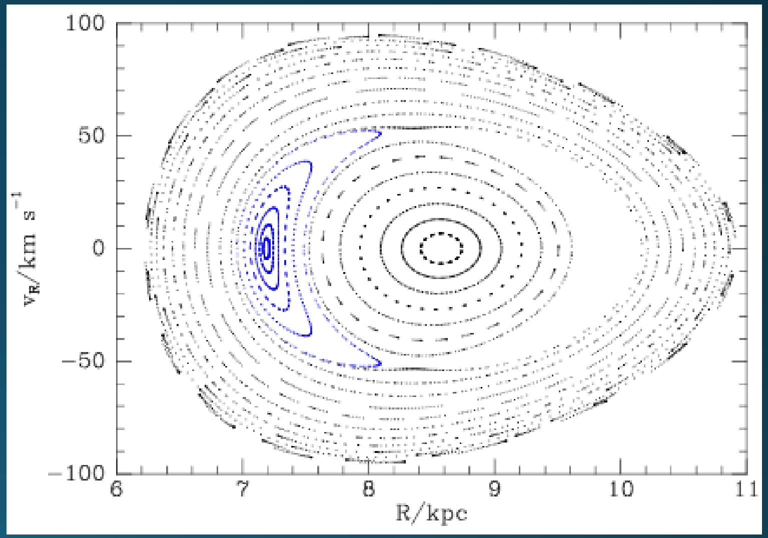
Resonant trapping

- Sometimes variance of H on image can't be driven to zero
- Torus with the specified actions doesn't exist
- But foliation by tori defines a Hamiltonian H'=<H> for which constructed tori are orbital tori
- So we have H' with AA vars plus perturbation Δ=H-H' and can apply Hamiltonian p-theory
- The problem will be that there's a `slow' angle θ_s=N.θ
- Make trivial canonical transformation so this becomes θ_1
- Fourier expand $\Delta(\mathbf{J},\boldsymbol{\theta}) = \sum_{\mathbf{n}} h_{\mathbf{n}}(\mathbf{J}) e^{i\mathbf{n}\cdot\boldsymbol{\theta}}$
- Neglect fast angles θ_2 & θ_3





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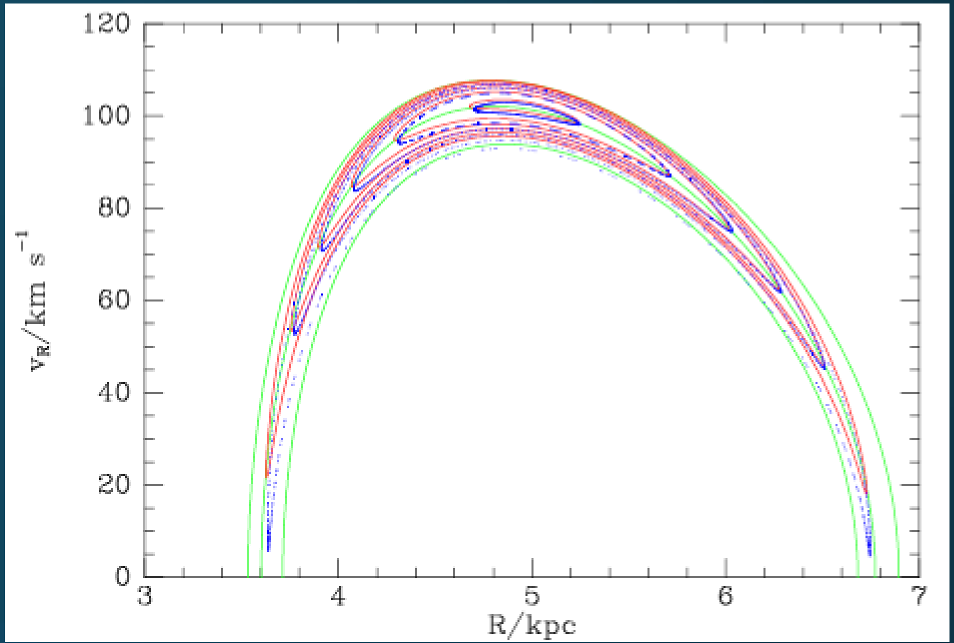
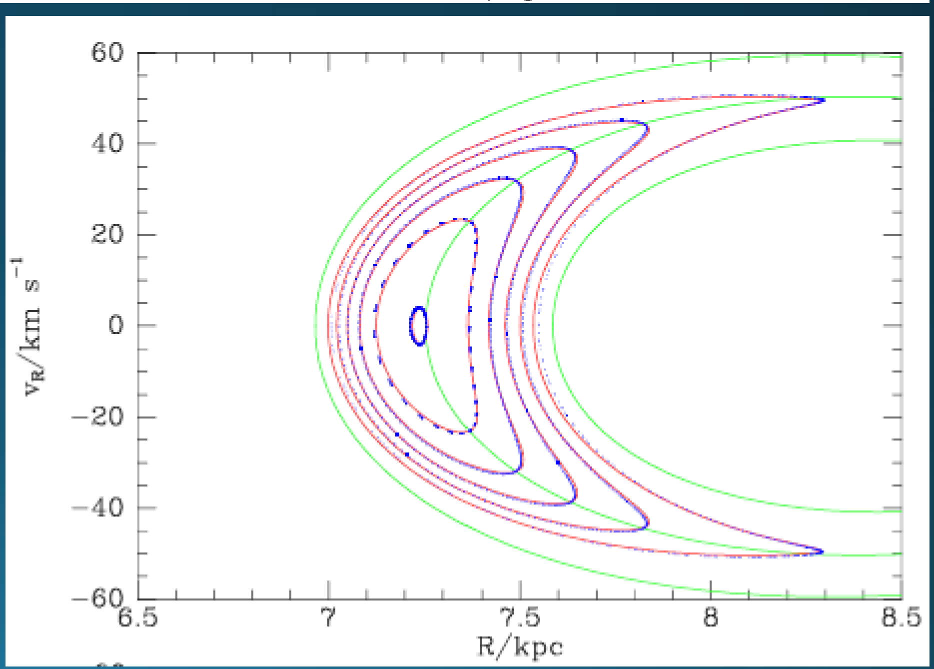
Resonant trapping

- Now have dynamics in (J_1,θ_1) plane with orbit Δ=const
- Δ often approximated by pendulum H but Kaasalainen (1996) showed you can do better
- 1d integrals now yield analytic model of tori, complete with AA vars
- Excellence of model demonstrated by Poincare SoSs
- Amazing fit quality!





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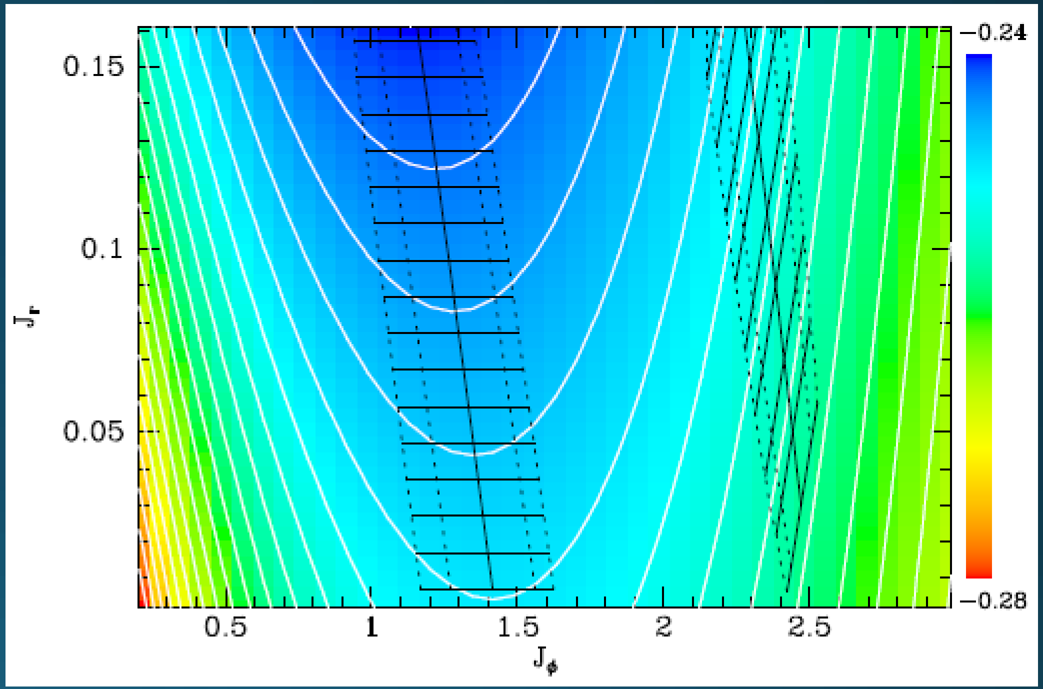
Non-axisymmetric tori

- Assume Φ(x,t) is stationary in steadily rotating frame
- then $H(\mathbf{x},\mathbf{p}) = H_{\rm in}(\mathbf{x},\mathbf{p}) - \omega_{\rm p} p_\phi$
- In axisymmetric Φ(x), $J_\phi = p_\phi$
- So H can be split in H(J) plus perturbation $H(\boldsymbol{\theta},\mathbf{J}) = \{\overline{H}(\mathbf{J}) - \omega_{\rm p} J_\phi\} + H_1(\boldsymbol{\theta},\mathbf{J})$
- We do p-theory using axisymmetric tori -> non-axisymmetric tori
- In realistic bar, regions trapped by CR & OLR are extensive



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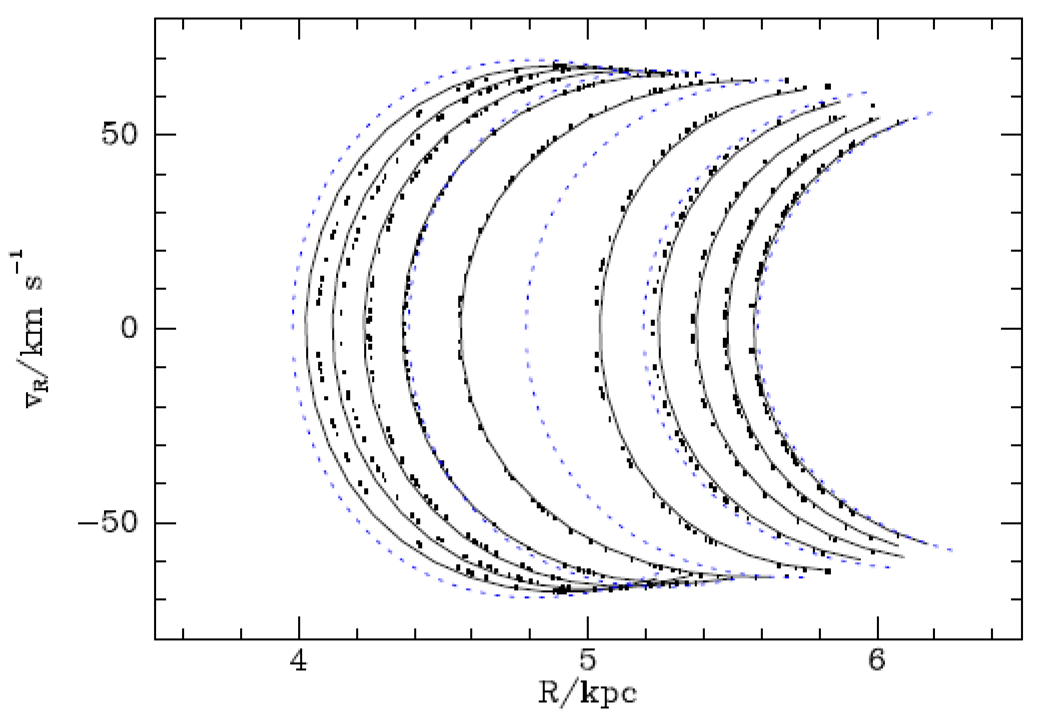
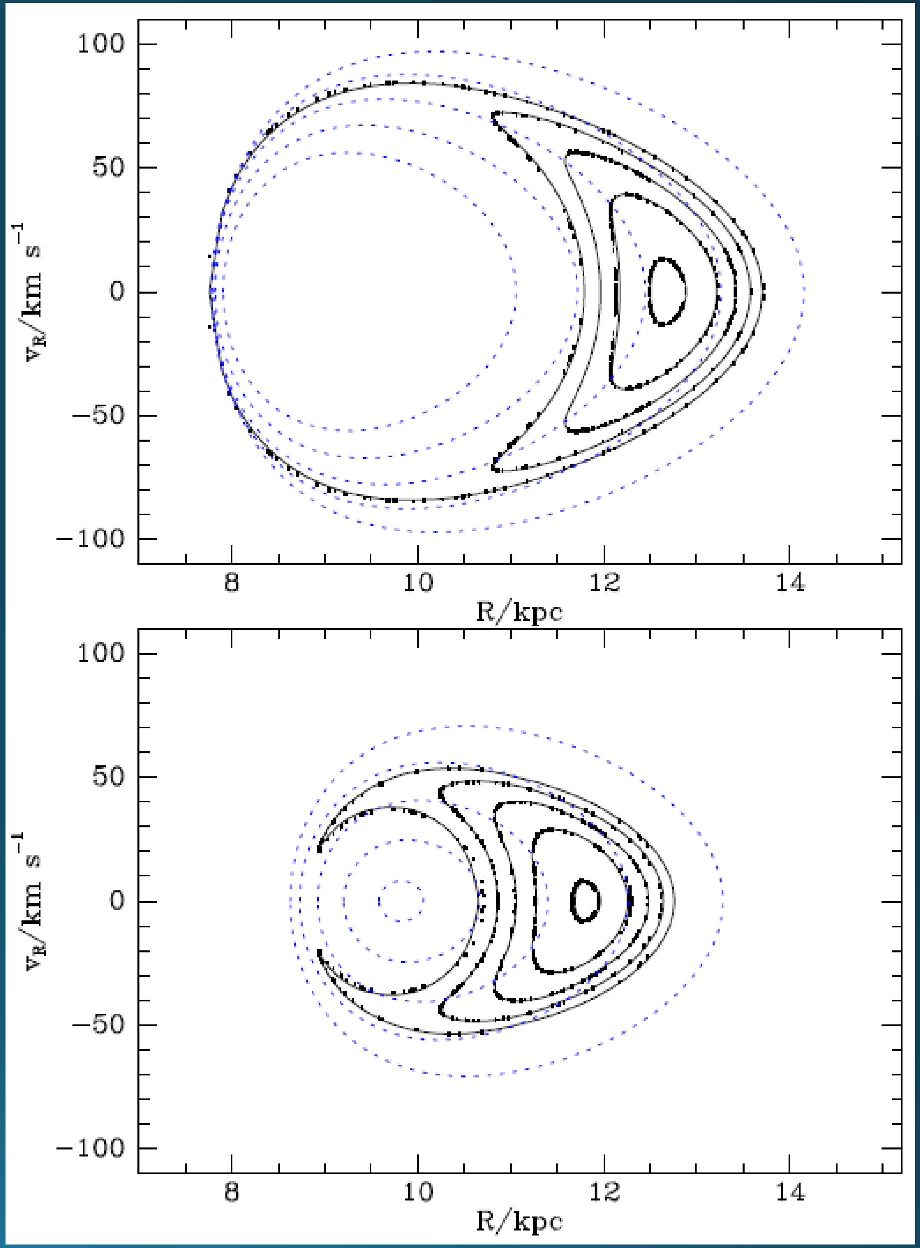
Trapped tori (B2018)

At CR Lz swaps between 2 values



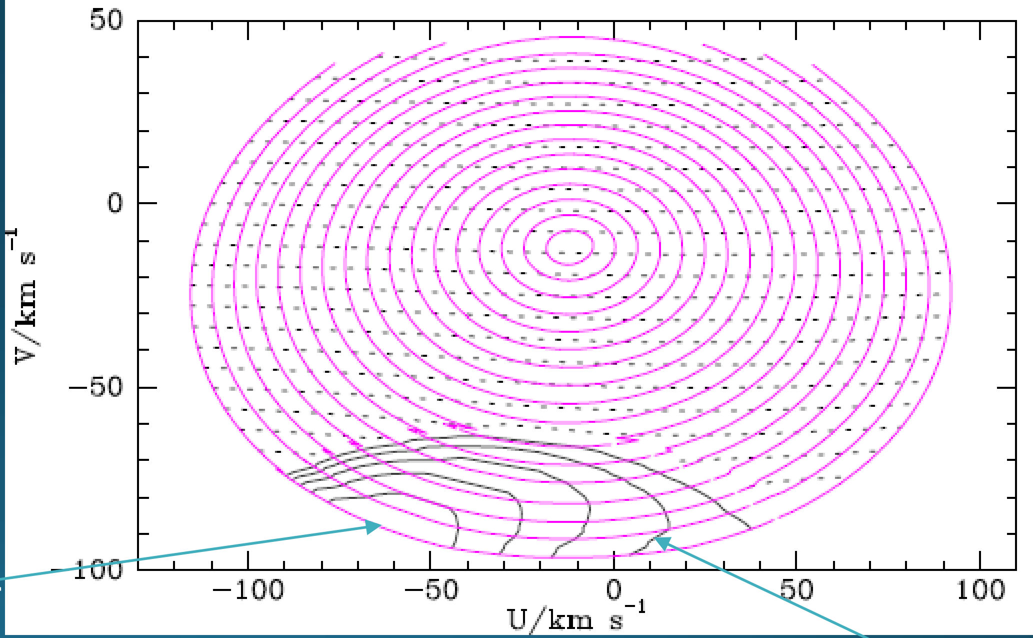
CR



OLR

Constants of motion

- Jeans thm -> f(J) -> need for J(x,v)
- We have AA vars for each non-axisymmetric torus
- Can plot contours of const J_i in (UV) plane
- f must be the same at different intersections of one pair contours
- From this can compute from tori ratio of densities in (U,V) at intersections
- Ratio of predicted to measured densities in (U,V) a constraint on Φ(x) additional to shape & location of trapped region



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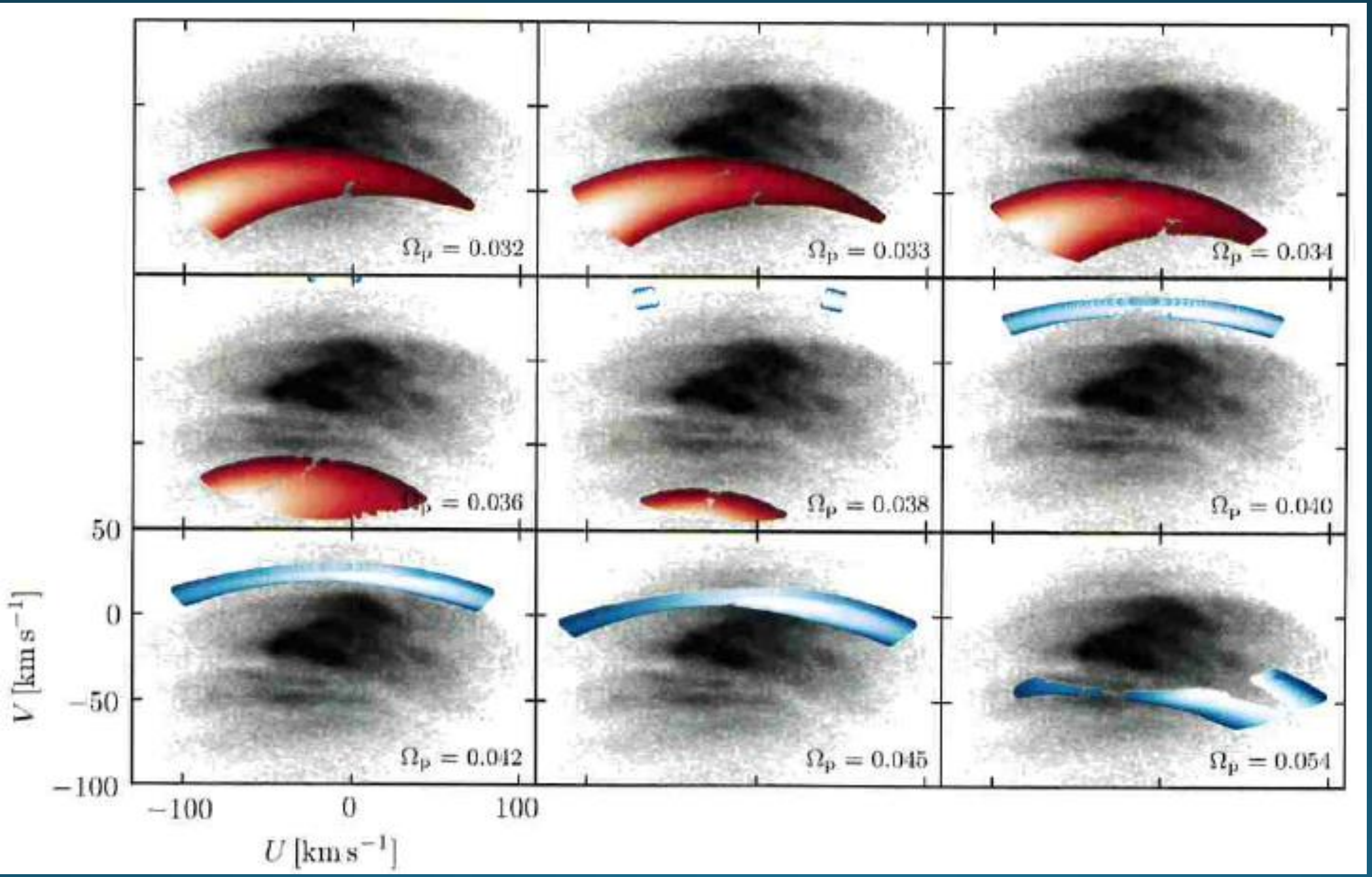
A remarkably low pattern speed?



B & Galligan in prep

About the DF

- The DF in the trapped zone is fundamentally unrelated to that outside it
- Since Dehnen (1999) it's been common to assign f by backwards integration to axisymmetric $\Phi(x)$ and f(J)
- This procedure is arbitrary
- Trapping is inherently non-adiabatic, so *does* depend on path taken by $\Phi(x,t)$
- We need to fit f(J) to data & recognise it constrains *history*
- In 2018 for fun I chose f(J) to minimise impact of trapping on (U,V)
- Nature hasn't been so perverse

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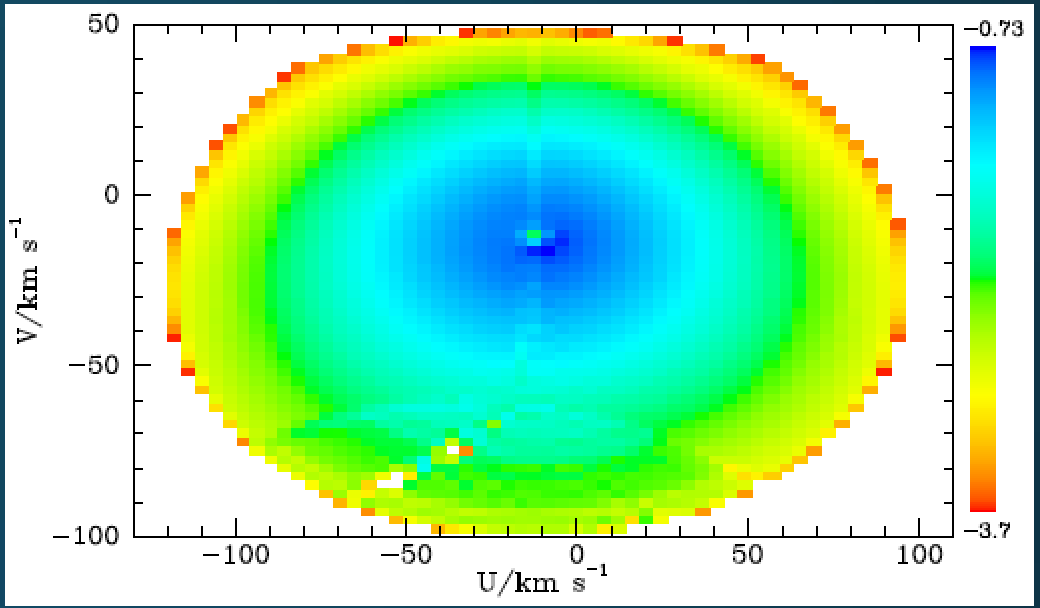

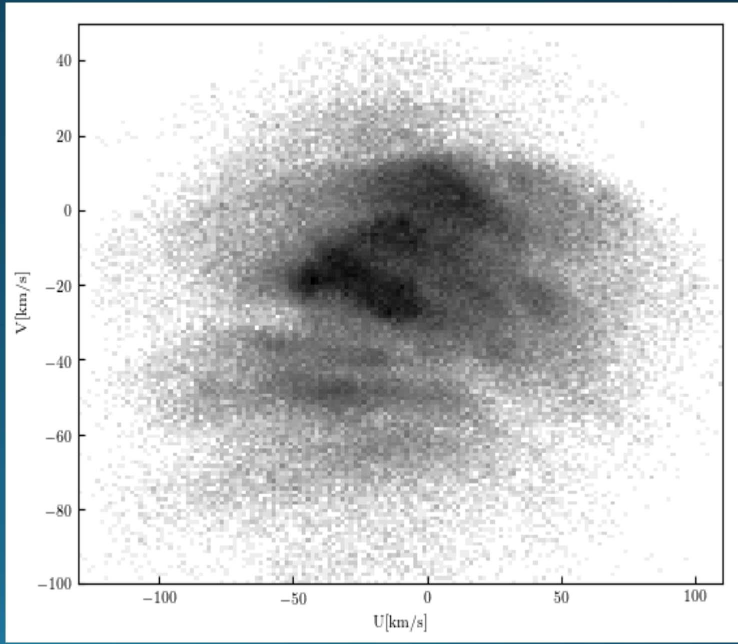

Gaia DR2 RVS

Conclusions

- Jeans thm is an enormously powerful tool
  - Our point of departure if at all possible
- AA vars are perfect for modelling both equilibria and disturbances
- Torus mapping + p-theory enable us to bring these tools to bear on the bar
- We have no reliable means of predicting DF for trapped orbits
- We have only scratched the surface
  - Vertical structure? J_z computed but seems to play minor role
  - Harmonics $m > 2$ – likely to introduce chaos
  - Comparison of predicted/measured densities still involves small numbers of particles